Power Allocation Schemes for Pilot Symbol Assisted Modulation over Rayleigh Fading Channels with no Feedback

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***Abstract*— In communication over time-varying Rayleigh fading channels, adaptive coded modulation for Pilot Symbol Assisted Modulation (PSAM) without feedback has been shown to yield significant benefits in terms of achievable rates [5]. This technique adapts transmission rate at the sender to the quality of the channel estimate at the receiver but keeps the mean power constant throughout. In this paper, we show that this adaptive PSAM scheme can be further improved if power and rate are jointly adapted to the quality of the measurement at the receiver. We study power distribution schemes that apply the following principle: more power is allocated to symbols corresponding to better estimates at the receiver, while maintaining the average energy constraint satisfied within a period. We find that this simple scheme performs better than adapting to channel quality using schemes akin to 'water filling'. Our model is a Rayleigh fading channel where time-variance is described by a Gauss-Markov model [4]. The transmitter periodically sends pilot tones to measure the channel at the receiver. We interleave different codes, while maintaining the power constant over a codebook, and the average power over codebooks satisfying the constraint. Performance is quantified in terms of achievable rates. Our scheme does not require any real time computation or adaptation at the transmitter, and so comes at no extra cost with respect to [5]. When considering causal and non causal estimation strategies at the receiver, considerable improvement was attained without any added complexity.**

I. Introduction

Adaptive schemes are a common means of allieviating the effect of channel time variation in wireless signal transmission. These schemes aim at improving the estimate of the signal at the receiver and adapting the transmission strategies of the sender, typically using feedback. If feedback is not available, another type of adaptive technique is still applicable, adaptive coded modulation for pilot symbol assisted modulation [5]. This modulation strategy does not use feedback to determine its transmission scheme, rather the sender uses the channel estimate quality at the receiver to modify transmission, where the measurement quality is quantified using the estimation error variance. Thus the transmission adapts its signaling and coding to the quality of the channel measurement, rather than to the quality of the channel . Pilot symbols, known for both the sender and the receiver, are sent at regular intervals T and are used to obtain a measurement of the channel. While in [5] the transmit energy is fixed over the interval and achievable rates are maximized, our approach is to maximize the rates while allowing the transmit power to vary over time.
It is shown in [5] that the achievable rates of this scheme are that of a Ricean channel where the transition probabilty distribution is known at the receiver. In effect, the estimation error is nothing but the Rayleigh component of the Ricean channel.

The channel model in [5] is as follows. A sender and receiver are connected by a time-varying Rayleigh fading channel, modeled by a Gauss Markov process [4]. The sender sends coded data, maintaining the average per symbol power constant. At regular intervals, pilot symbols, with energy equal to the average energy constraint, are sent for channel measurement (see Fig. 1).

We consider binary signaling, because of its near optimal performance in terms of channel capacity at low SNRs; and our goal becomes that of maximizing mutual information between input and output for a given channel and energy constraints. This mutual information represents feasible achievable rate.
At the receiver, two estimation strategies are analysed,

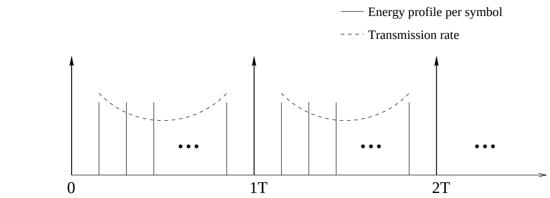

Fig. 1. Schematic representation of Adaptive Coding for PSAM, E(1,1) [5]

one causal and one noncausal. The causal estimation scheme denoted E(1,0) is a One Pilot based Estimation. The receiver estimates the channel using just the previous pilot tone. The second case is a noncausal one, E(1,1); channel estimation is based upon the most recently sent pilot and the next one. The analysis of this rate adaptation strategy shows that an improvement of up to 30% in achievable rates can be attained by optimizing spacing between pilot symbols [5].

The gist of the approach in [5] is adaptation to the fact that channel estimation quality improves with proximity to the nearest pilot tone. As seen in Fig. 1, while in [5] the strategy adapts transmission rate to the distance of a symbol to the nearest pilot, it does not do power adaptation. From a theoretical point of view, when dealing with time-varying channels, capacity achieving schemes generally do not employ flat power. While most work in the litterature adapts power to Signal to Noise Ratio (SNR) using feedback, to our knowledge, adapting power to the quality of channel measurement has not yet been investigated.

In this study, we follow the channel model in [5] to investigate to what extent adaptive power modulation, in addition to rate adaptation can improve capacity. We show that varying the energy of the modulated signal according to its distance to the closest pilot is beneficial. Because our scheme does not require feedback, it does not use real-time computation. Thus, the adaptation can be designed offline and applied in a periodic fashion. Hence, this benefit comes at no extra cost with respect to those of [5] and is thus a free improvement.

The channel we consider is not a block fading one as in [1] but a continuously time-varying channel. Also note that we do not attempt to optimize power for sounding, as is done in [3], but rather look into power allocation for a given energy constraint. Figures 2 and 3 are schematic representations of the optimum energy profile we found in our study. These will be explained further at a later stage.

In Section 2, we will give a brief review of adaptive coded modulation for PSAM [5] presenting the model and the different results obtained for the two estimation

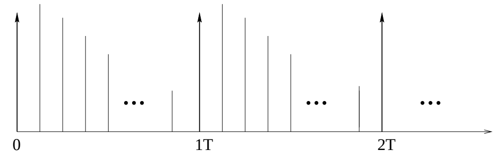

Fig. 2. Power Allocation for E(1,0)

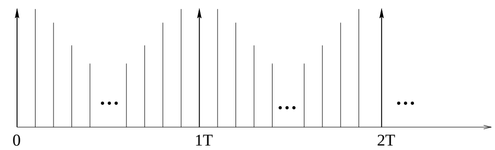

Fig. 3. Power Allocation for E(1,1)

strategies. In Section 3, we describe the power allocation schemes used for the One Pilot and Two Pilot Estimations. In Section 4 we present our numerical results and conclude in Section 5.

II. Adaptive Coded Modulation for Pilot Symbol Assisted Modulation

The discrete time model is a Rayleigh fading channel

$$Y_k = R_k X_k + N_k,$$

where X_k is the input and Y_k the output at sample time k. The noise term N_k is Gaussian with mean 0 and variance σ_N^2. The fading coefficent R_k is also a zero-mean Gaussian random variable with variance σ_R^2. Since the process is considered to be a Gauss-Markov one, fading coefficients are correlated by

$$R_k = \alpha R_{k-1} + Z_k.$$

The parameter α, which determines the time variation of the channel [4], is positive and the $\{Z_k\}$'s are IID circular Gaussian distributed with mean 0 and variance $(1-|\alpha|^2)\sigma_R^2$. We impose an average power constraint P on the input.

In this scheme, pilot symbols have energy equal to the square root of the given power. At sounding times, which are multiples of T,

$$Y_{lT} = R_{lT}\sqrt{P} + N_{lT}. \tag{1}$$

Note that we do not seek to optimize the power P, but rather maximize the achievable rate for a given P. Based on the $\{y_{lT}\}'s$, the fading coefficients are estimated at each time step using a standard Bayesian Least Square Estimation. The transition probability density, [5], becomes

$$p_{Y_k|Xk}(y_k|x_k) = \frac{1}{\pi(v_k|x_k|^2+\sigma_N^2)} \exp\left\{\frac{-|y_k-\hat{R}_k x_k|^2}{v_k|x_k|^2+\sigma_N^2}\right\},$$

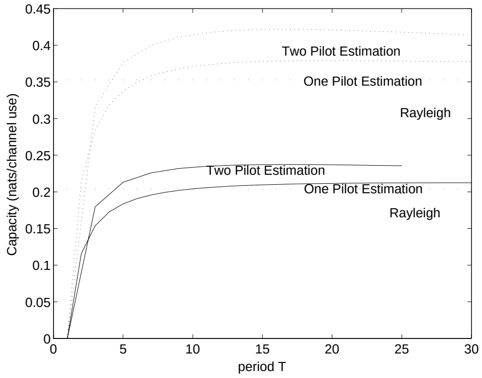

Fig. 4. Performance comparison of causal and non causal estimation methods, $\alpha = 0.99$, SNR=0dB (full) 5dB (dotted) [5]

where $\hat{R}_k$ is the estimate of R_k and v_k its error variance. Therefore, the mutual information at a given time:

$$\begin{aligned} &I_k(X_k; Y_k|\{Y_{lT}\} = \{y_{lT}\}) \\ =&p_k(1)\int p_{Y_k|X_k}(y|x_k(1)) ln \frac{p_{Y_k|X_k}(y|x_k(1))}{p_{Y_k}(y)} dy \\ +&(1-p_k(1))\int p_{Y_k|X_k}(y|x_k(2)) ln \frac{p_{Y_k|X_k}(y|x_k(2))}{p_{Y_k}(y)} dy. \end{aligned}$$

and, the achievable rates are

$$\begin{aligned} &E\left[\frac{1}{T}\sum_{k=lT+1}^{(l+1)T-1} I_k(X_k; Y_k|\{Y_{lT}\} = \{y_{lT}\})\right] \\ &= \frac{1}{T}\sum_{k=lT+1}^{(l+1)T-1} E\left[I_k(X_k; Y_k|\{Y_{lT}\} = \{y_{lT}\})\right]. \end{aligned}$$

We maximize the sum of the expected value of the mutual information [2] using numerical techniques [6] in *Matlab* because there is no closed-form solution for the expression of the optimal input distribution and power allocation. The analysis is done on the two above mentioned estimation methods (E(1,0), E(1,1)) for SNRs of 0 and 5dB and $\alpha = 0.99$. The performance of the different systems is summarized in Fig. 4.

III. Power allocation schemes

The same channel model is considered as in [5]. The only difference is that data symbols no longer satisfy the average energy constraint individually, but collectively. By using the optimizing tools of *Matlab*, we attempt to find the optimal power distribution among the data symbols, with the average energy of these being equal to the energy constraint. Note from (1) that we maintain $\sqrt{P}$ fixed and do not adapt the power in the pilots.

A. One Pilot based causal Estimation: E(1,0)

This method uses the most recently sent pilot tone to estimate the fading coefficients (by least square estimation). The estimates of the fading coefficients and the variance of the error were found in [5] to be:

$$\begin{aligned} \hat{R}_k &= \frac{\sqrt{P}\sigma_R^2}{P\sigma_R^2 + \sigma_N^2}\alpha^{(k-lT)} y_{lT}, for lT < k < (l+1)T \\ v_k &= \sigma_R^2 - \frac{P\sigma_R^4}{P\sigma_R^2 + \sigma_N^2}|\alpha^{(k-lT)}|^2. \end{aligned}$$

We show that, the optimal power allocation over the data symbols has a decreasing character with respect to the distance to the last sent pilot (see Fig. 2) and improves significantly the rates over the scheme where no power adaptation is performed.

B. Two Pilots based noncausal Estimation: E(1,1)

Here, we do our estimation using the most recently sent pilot tone as well as the one transmitted next. In this case:

$$\begin{aligned} \hat{R}_k &= \frac{\sqrt{P}\sigma_R^2}{(P\sigma_R^2 + \sigma_N^2)^2 - P^2\alpha^{2T}}[\alpha^{(k-lT)}(P\sigma_R^2 + \sigma_N^2) \\ &\quad -P\alpha^{2T-(k-lT)}]y_{lT}] + [\alpha^{T-(k-lT)}(P\sigma_R^2 + \sigma_N^2) \\ &\quad -P\alpha^{T+(k-lT)}]y_{(l+1)T}], with \\ v_k &= \sigma_R^2 - \frac{P\sigma_R^4}{(P\sigma_R^2 + \sigma_N^2)^2 - P^2\alpha^{2T}}[(P\sigma_R^2 + \sigma_N^2) \\ &\quad [\alpha^{2(k-lT)} + \alpha^{2[T-(k-lT)]}] - 2P\alpha^{2T}. \end{aligned}$$

The optimal power distribution among the data symbols was found numerically to look like a convexe parabol (see Fig. 3). For any period T, between two pilot symbols, the power levels decrease as you move inwards from the pilot tones to the middle.

Note that, while it may seem natural to investigate 'water filling' type techniques to adapt transmission to channel estimation error, we found that this scheme is not appropriate. Because of constraints of space, the plots are not shown here.

IV. Numerical Results

In the folowing plots, we show the achievable rates with respect to different values of the period T. The dotted curves correspond to the values calculated in [5], meaning the scheme adapts rates to the location from a coded symbol to pilot symbols. The full curves correspond to input distributions that adapt both power and rate to the location from a coded symbol to pilot tones. Thus, the closer a symbol is to a pilot tone, the higher the rate *and* power it is coded with. Of course,

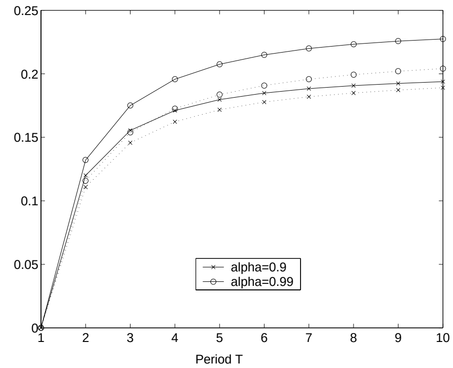

Fig. 5. Comparison of E(1,0) with (full) and without (dotted) power adaptation. $\alpha = 0.9$ and 0.99; SNR=0dB

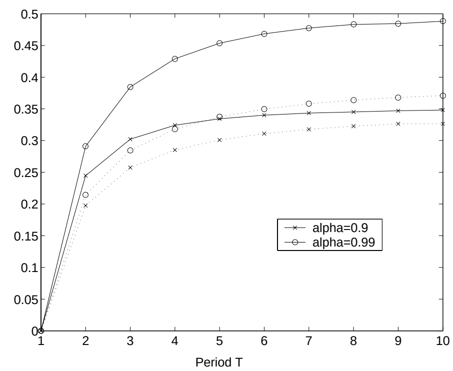

Fig. 6. Comparison of E(1,0) with (full) and without (dotted) power adaptation. $\alpha = 0.9$ and 0.99; SNR=5dB

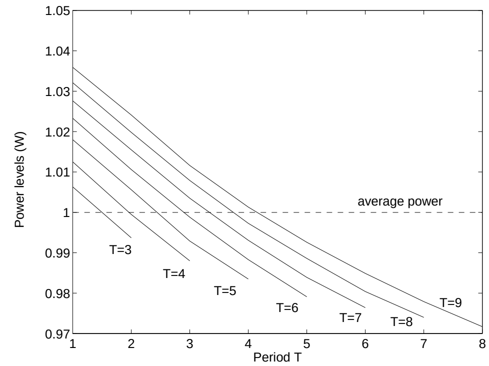

Fig. 7. E(1,0) Power levels for $\alpha = 0.99$, SNR=0dB

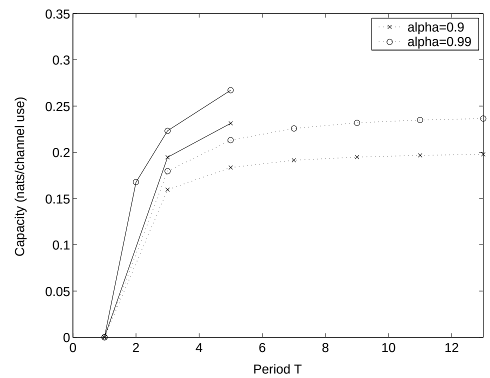

Fig. 8. Comparison of E(1,1) with(full) and without(dotted) power optimization, $\alpha = 0.9$ and 0.99, SNR=0dB

within a period, the average power of the data symbols is equal to the power constraint. As in [5], the scheme is implemented using inteleaved codes with T being the interleaving period. The results are computed for $\alpha = 0.9, 0.99$ for SNRs of 0 and 5dB. The optimal value of a scheme occurs at the maximum of the curve.

Figures 5 and 6 compare the performance of E(1,0) $with$ power adaptation and $without$ [5] for both values of α for SNRs of 0 and 5dB respectively. In these plotS, the full curves correspond to achievable rates with power and rate adaptation and the dotted ones to those with just rate adaptation [5].

We can see the benefit of power adaptation is accentuated for high SNR and high values of α. For instance, for $T = 10$, $\alpha = 0.99$ and SNR=5dB, performing power adaptation $inaddition$ to rate adaptation yields an improvement of more than 31%!

Figure 7 shows the optimum power distribution versus time found for E(1,0) for different values of T. The range of each line is T-1. The power profile is calculated for $\alpha = 0.99$ and an SNR of 0dB.

Hence, the optimum power distribution for E(1,0) was found to be decreasing with respect to the distance to the last sent pilot. Also, as we increase T, the quality of estimation is decreasing, and so we need to start with larger value of the power (see Fig 6).

For E(1,1), Figures 8 and 9 compare achievable rates with and without power optimization for both values of α (0.9 and 0.99) for SNRs of 0 and 5dB respectively.

Again, the benefits of performing power adaptation considerable, and mostly for high SNRs. As for the two estimation methods, comparing the plots for E(1,0) and E(1,1) shows us that doing power adaptation yields more improvement with two pilots than one pilot estimation.

The optimum power distribution found for two pilots based estimation is shown in Fig. 10 for $\alpha = 0.99$ and SNR=0dB. Indeed, the optimum distribution allocates more power as the distance of symbols to the nearest

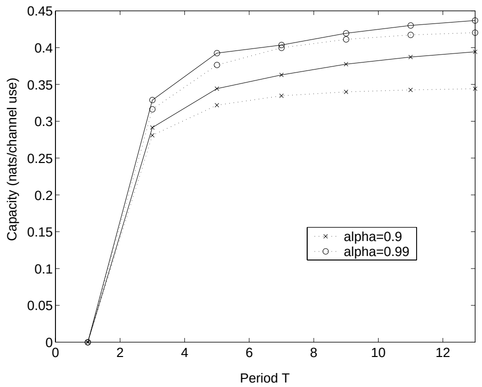

Fig. 9. Comparison of E(1,1) with(full) and without(dotted) power optimization, $\alpha = 0.9$ and 0.99, SNR=5dB

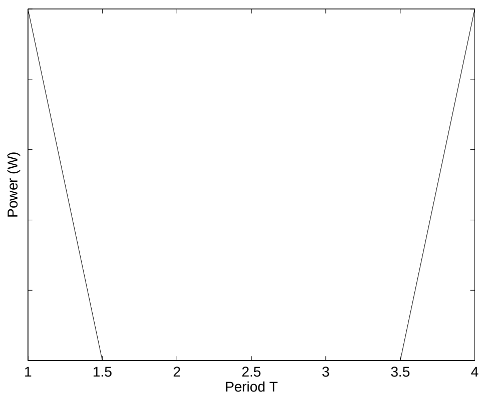

Fig. 10. E(1,1) Power levels for SNR=0dB, $\alpha = 0.99$

pilot tone decreases.

Note that,

V. Conclusion

Building on adaptive coding for pilot symbol assisted modulation without feedback, we investigated how this adaptive scheme can be further improved if power and rate are jointly adapted to the quality of the measurement of the receiver. We have shown that while keeping the average power of coded symbols constant within a period, allocating power according to a symbol's distance from pilot tones can yield significant improvement in achievable rates. We studied power adaptation for both one and two pilot-based estimation strategies at the receiver and found the optimal power distribution: symbols closer to pilot symbols have more power and higher rates allocated to them, while further away symbols are given less power and rates. This improvement comes at no extra cost in term of computation or adaptation at the sender, requires no real-time computation and hence adds no complexity to the problem.

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